Design and construction of long-span post-tensioned tubular steel structures

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ABSTRACT: The design of long-span post-tensioned tubular steel structures in which steel cables are housed within the hollow profiles is outlined from fundamental principles. Treatment of the structural form as a planar catenary is discussed and then extended to three dimensional shell structures. The effect of post-tensioning on the behaviour of individual members is examined through numerical and analytical modelling. The influence of the bonding material between the post-tensioned cable and tubular casing is discussed, since this is crucial to the performance of these structures. The construction methodology is outlined and examples of implemented structures spanning up to 120 m are presented.

1 INTRODUCTION

The current drive towards sustainable development now places a high level of importance on the reduction of embodied energy within the construction industry. The World Steel Association (WSA) has identified that steel production accounts for 3-4% of the World’s greenhouse gas emissions and that 1.8 tonnes of CO\textsubscript{2} are emitted for every tonne of steel produced (WSA, 2011). Post-tensioned tubular steel structures, which have the ability to store energy, under increasing pre-load can achieve reduced material consumption and hence a greatly reduced their carbon footprint. The development of such structural solutions is outlined in the current paper.

This paper builds upon the fundamental principles of the equilibrium of arches and shows how these principles may be applied to post-tensioned steel structures. Post-tensioning of cables and cable net structures that are integrated into the structural form generates a preload condition that controls deflection. Controlling deflection in this manner substantially reduces the amount of material required in the construction of long-spanning structures. Tubular elements are fundamental to this form of construction since the post-tensioning cables are housed within the structural profiles.

The design solution of two dimensional post-tensioned arches is firstly investigated, followed by the application of post-tensioning technology to three dimensional shell structures.

2 FUNDAMENTAL PRINCIPLES

One of the main factors that determines the maximum load carrying capacity of an arch is its shape (O’Dwyer, 1999). Robert Hooke was the first philosopher to identify that the most efficient shape of an arch can be derived from the loads acting upon it. In 1675, he presented his solution to the Royal Society in the form of an anagram, ‘As hangs a flexible line, so inverted will stand the rigid arch’ (Huerta, 2006). Antonio Gaudi, some 200 years later became perhaps the best known exponent of this philosophy by creating hanging chains with lead shot masses contained in bags to simulate the weight of masonry (Huerta, 2006). The shape created by the chain could then be inverted to generate the line of thrust along which compressive forces are transmitted to the ground. Ingeniously, he placed a mirror under the hanging chain model to construct the arches of the Colonia Gueli Church (Huerta, 2006).

Various methods of obtaining the thrust line have been proposed, but one of the most commonly adopted methods is the graphical method of the funicular polygon (Heyman, 1982). The solution is obtained by constructing a force polygon of all the forces acting on the structure. Figure 1(a) shows the structure under consideration. Figure 1(b) shows the force polygon diagram, with the position of the centre O representing the unknown horizontal reaction. The lines radiating from this centre indicate the inclinations of the weightless chain at various sections, as illustrated in Figure 1(c). Finally, using Hooke’s analogy, the hanging chain is inverted to give the thrust
line shown in Figure 1(d). Since the horizontal reaction force can take any value, an infinite number of thrust lines can be generated for the applied loading (Heyman, 1982); the larger the horizontal reaction, the shallower the arch.

Since an arch provides support to the applied loads through compression, masonry and concrete were the original materials of choice for the construction of arches. However, owing to the relative ease and reduced construction time associated with steel structures, growing interest is now placed on the use of steel in the construction of arches (Nazir, 2003).

Thrust line analysis is a powerful tool that can be used to design a wide range of long-spanning steel structures. Figure 2 illustrates how thrust line analysis may be applied to the design of a portal frame for an aircraft hangar.

The distributed uplift wind load in Figure 2 is applied to a weightless chain of a chosen length; using the funicular polygon method discussed earlier, the shape and tension within the chain is determined. The length of the chain is typically chosen in order to ensure that the bottom chord of the frame is always under tension during wind uplift. Inverting the resultant hanging chain gives the thrust line. It should be noted that consideration of other load cases would result in the formation of different thrust lines, from which an average thrust line can be chosen for design.

If the structure were to be built along the thrust line, it would not be subjected to bending and thus a minimum amount of material would be required to support the design loads. However, structures designed in this way, while minimising the required material, may not provide the desired space. Instead, structures, such as the hangar in Figure 2, are built to satisfy the spatial requirements by designing against the bending moments that would arise due to the lever arm between the structure and the thrust line.
3 POST-TENSIONING

Application of the post-tensioning technique to the construction of trusses is described in this section. The post-tensioned cables are located within the bottom chord of the tubular trusses and apply a compressive force to the chord members, which is opposite in nature to the resultant forces arising from the externally applied gravity loads. The capacity of the truss is maximized since the tensile strength of the lower chord is only utilized after the compressive force induced by the post-tensioned cables has been exceeded.

3.1 Construction methodology

As with all construction, it is imperative that the load cases which arise during assembly are given due consideration; this is especially important in this type of structure since a significant amount of force is applied to the elements through post-tensioning of the cables. Furthermore, from practical experience, designing the buildability of a structure into the architecture is vital for achieving a successful outcome. With this in mind, the following procedure has been carefully devised for the construction of post-tensioned steel frames (Ellen, 1987).

1. Construct the tubular trussed main span and connecting columns at ground level.

2. Post-tension the cables encased within the bottom chord of the centre span to obtain the required shape. The segmental length of the lower chord (i.e. the distance between the vertical/diagonal members) determines the curvature of the upper and lower chords; therefore the segmental length may be varied at the design stage to obtain the desired shape.

3. Assemble the remaining column structure to form the complete frame, connecting the bottom chord cables of the centre span to those of the column structure and post-tensioning them further. The void space between the cables and the tube is then grouted with cementitious material such as Portland cement with chemical additives.

The ability to control deflections is at the heart of a successful solution to long-spanning steel structures. Figure 4 illustrates the pre-load condition generated due to the introduction of post-tensioned cables in the bottom chord. The resultant pre-load condition, shown by the inward pointing arrows, provides resistance against deflection as well as wind uplift. An 84 m span example of such a structure with an 84 m span is shown in Figure 5, while a smaller 25 m span temporary structure being shown in Figure 6.
3.2 Behaviour of post-tensioned elements

A study has been conducted to investigate the behaviour of individual post-tensioned tubular members. The aim was to establish the effect of the bonding material and pre-load on the stiffness and axial load carrying capacity. Figure 7 shows a numerical simulation of a segment of a post-tensioned chord. The chord, which is a Grade 250 circular hollow section, is bonded to the post-tensioned cables using high strength grout. The properties of the tube and cable are given in Table 1.

![Tubular element, Cable, Bonding material](image)

**Figure 7 Post-tensioned tubular element**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length ( L_0 )</td>
<td>1000 mm</td>
</tr>
<tr>
<td>Cross-sectional area of cable</td>
<td>120 mm(^2)</td>
</tr>
<tr>
<td>Cross-sectional area of tube</td>
<td>750 mm(^2)</td>
</tr>
<tr>
<td>Young’s modulus of cable and tube</td>
<td>210 000 N/mm(^2)</td>
</tr>
<tr>
<td>Yield strength of tube ( f_y )</td>
<td>250 N/mm(^2)</td>
</tr>
<tr>
<td>Yielded strength of cable ( f_{cy} )</td>
<td>2100 N/mm(^2)</td>
</tr>
</tbody>
</table>

Table 1. Properties of elements of the numerical model.

Figure 8 shows the numerical results when tensile strain is applied to a chord member with varying levels of post-tension forces within the embedded cables. As the graph demonstrates, the axial load carrying capacity is substantially increased when post-tension is applied to the cables. This can be attributed to the fact that the compressive force imposed by the post-tensioning of the cables must first be overcome before the chord member begins to utilize its tensile strength.

![Graph showing load vs displacement](image)

**Figure 8. Numerical results for axial load carrying capacity of a post-tensioned chord member.**

Figure 8 shows that the magnitude of the post-tension affects not only the ultimate axial load carrying capacity but also initial axial stiffness prior to yielding.

Experimental studies are planned to investigate this behaviour and to assess the influence of the grouting material.

An analytical model was also devised to represent the behaviour of post-tensioned elements by considering the different stages of the post-tensioning and subsequent load application processes. The following points describe how an expression for the relationship between axial load and displacement is obtained for the case where the tube yields first.

**Post-tensioning:**

**Stage 1: Initial position**

Initially, the length of the tube \( L_t \) and the cable \( L_c \) are equal, as illustrated on Figure 9, hence:

\[
L_t = L_c = L_0
\]

![Diagram showing initial position](image)

**Figure 9. Dimensions of chord before post-tensioning**

**Stage 2: After post-tensioning**

After post-tensioning, the cable is in tension and tube is in compression, with the system self-equilibrating under internal forces \( P \).

![Diagram showing after post-tensioning](image)

**Figure 10(a). Post-tensioned cable (b) Compressed tube. (c) Dimensions of tube and cable after post-tensioning.**

From Figure 10(c), the lengths of the tube and cable after post-tensioning are given by Eqs (2) and (3) respectively, in which \( x_t \) is the displacement of the compressed tube and \( x_c \) is the extension of the compressed cable.
Stage 1: $L_1 \leq L_o$

As Figure 11 illustrates, in this region of axial strain, the compressed tube unloads to its original length whilst the cable is being stretched further.

Assuming that the tube is not compressed beyond its yield load, the axial displacement $x_1$ and axial force $N_1$ when the tube fully unloads ($L_i = L_o$) can be expressed by Eqs (4) and (5) respectively.

$$x_1 = x_i = \frac{PL_o}{A_i E}$$  \hspace{1cm} (4)

$$N_1 = K_c (x_c + x_i)$$  \hspace{1cm} (5)

where $K_c = \frac{A_c E}{L_o}$

Stage 2: $L_o \leq L_i \leq L_{cy}$

For this range of axial displacements, shown in Figure 12, both the cable and tube are in tension.

When the tube yields in tension the axial displacement $x_2$ and axial force $N_2$ are given by Eqs (6) and (7) respectively.

$$x_2 = x_1 + x_{ty} = x_1 + \frac{f_{ty}L_o}{E}$$  \hspace{1cm} (6)

$$N_2 = N_1 + (K_c + K_i) \frac{f_{ty}L_o}{E}$$  \hspace{1cm} (7)

where $K_i = \frac{A_i E}{L_o}$

Stage 3: $L_{ty} \leq L_i \leq L_{cy}$

In this region of axial displacement the tube is strain hardening whilst the cable utilises its elastic axial stiffness to resist tensile loading. When the cable yields in tension the axial displacement $x_3$ and axial force $N_3$ are given by Eqs (8) and (9) respectively.

$$x_3 = x_{cy} - x_c = \frac{f_{cy} L_o}{E} - \frac{PL_o}{A_i E} \hspace{1cm} (8)$$

$$N_3 = A_c f_{cy} + A_i f_{ty} + K_{c,st} (x_3 - x_2) \hspace{1cm} (9)$$

where $K_{c,st}$ is the strain hardening stiffness of the cable.

Stage 4: $L_i \geq L_{cy}$

Both the cable and the tube will be strain hardening for axial displacements $x$ in this range; the corresponding axial force $N_4$ is given by Eq (10).

$$N_4 (x) = K_a (x - x_4) + N_3 \hspace{1cm} (10)$$

where $K_a$ is the strain hardening stiffness of cable and tube.

The obtained analytical results, using the properties of Table 1, are compared in Figure 13 with the numerical results of Figure 8 showing a close match. It should be noted that a drop in stiffness due to the yielding of the tube is usually greater than the case when the cable yields first since axial stiffness is directly proportional to the cross-sectional area and the cross-sectional area of the tube is typically larger than the cross-sectional area of the cables.

Figure 11. Tensile loading prior to initial pre-compression in tube being overcome.

Figure 12. Tensile loading after cable fully unloads initial compression.

Figure 13. Comparison between analytical and numerical results for the axial response of a post-tensioned chord member.
To gain further insight into the behaviour of post-tensioned chord members, the parameters that define the axial stiffness of the cable and the tube were altered. The material properties given in Table 1 and post-tensioning values as shown in Figure 13 were used as a control for this investigation. The results shown in Figure 14 are for the case in which the Young’s modulus of the cable is halved from the value given in Table 1. In Figure 15, the cross-sectional areas of the tube and the cable were chosen such that the ultimate yield load, defined herein as the load at which both the tube and the cable yield, is kept constant while the ratio of cable to tube cross-sectional area is altered.

Figure 14 shows that when the Young’s modulus of the cable is reduced the tube yields at a lower load and that the stiffness drop after the tube yields is more significant.

3.3 Reverse loading
For structures where the dominant loads are downward, the chain model can again be inverted to find the optimal curvature to resist vertical loading through tensile action. Figure 16, which is part of the roof structure of the Sydney Olympic Stadium, illustrates how the vertical force may be transmitted to the supports through tensile action. Application of post-tensioned cables within the tubular bottom chord can again further enhance the structural system.

Figure 17. Hanging net forms the thrust surface for a shell when inverted, in similar manner to Hooke’s inverted chain.

3.4 Application to shells and other structures
Through the extension of the inverted chain philosophy, solutions to three dimensional shell structures can also be achieved. Figure 17 illustrates, just as in the hanging chain, how a hanging net adopts a shape when subjected to its self-weight and perhaps some wind loading. Following Hooke’s principle, the net could then be inverted to give the shape of the shell which can efficiently support the applied load. However, unlike the flexible chain, the hanging flexible net has multiple thrust paths which enable it to carry a wider range of load without changing its shape (Heyman, 1977). And thus the inverted shape can support significant amount of loading without incurring significant bending in any part.
The thrust line analysis of structures becomes complicated when three dimensional forces are involved such as in the case of shell structures (O’Dwyer, 1999). Several attempts have been made to determine the stability of masonry shells which require the thrust surface to be contained within the thickness of the shell since the material used in masonry construction such as stones and concrete have negligible tensile strength. One of the simplest, but also safe, approaches that can be used in the analysis of shell structures is a slicing technique. The analysis checks the overall stability of the masonry shell by dividing it into discrete sections and checking their equilibrium (Heyman, 1977). A more complex analysis of masonry shells which is attracting increasing interest is the force network model. Just as in the funicular polygon method for the analysis of arches, this model discretizes the loads. However, for shells an optimization procedure is required to determine the thrust surface and the collapse load factor. (O’Dwyer, 1999)

Reducing force equilibrium to a two dimensional problem, as in the slicing technique described earlier, the first author has designed and constructed several shell structures. Figure 18 shows an aircraft facility with shell type roofing that was constructed using intersecting truss arches which were designed using the post-tensioning technology described earlier.

3.5 Comparison with conventional methods

Owing to the relatively large structural members associated with long-spanning steel structures, the ease of transportation of structural components during the assembly of the structure is important. The post-tensioning technique reduces the total weight of the structure considerably, thereby minimising the time and cost spent on erecting the structure using cranes. Figure 19 shows a 78 m span post-tensioned portal frame being erected using just two cranes. The innovative solution led to an award, from the Australian Steel Institute.

![Figure 18. Aircraft hangar with a shell type roofing constructed using intersecting tubular arches.](image1)

![Figure 19. Award winning post-tensioned aviation hangar with a 78 m clear span erected with minimal assembly time.](image2)

![Figure 20. Erecting a shell structure made using post-tensioned LSB members without cranes.](image3)

![Figure 21. Illustrative comparison between conventional construction method and the new post-tensioning technology.](image4)
4 CONCLUSIONS

The addition of post-tensioned cables housed within the tubular chords of steel trusses has been shown herein to offer effective and efficient structural solutions. Application of thrust line analysis to the design of planar frames and three dimensional shell structures is discussed. The funicular force polygon diagram method was found to be a suitable means of obtaining the thrust line of tubular trussed arches. By designing shell structures with intersecting vaults, the three dimensional problem can be simplified using the slicing technique.

Post-tensioned cables induce compressive forces in tubular chord members which are opposite in nature to the forces caused by the imposed loading; the preload must first be overcome before the tensile capacity of the tubular chord members is utilised, enabling lighter sections to be used. This behaviour was demonstrated by means of numerical and analytical models.

A series of examples for a range of structures from around the world have been presented to demonstrate successful application of the post-tensioning technology.

5 REFERENCES

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